The text contains seven chapters. The first is an introduction, providing required definitions of terms and notation, and also including a survey of results that follow in the remaining chapters. This survey gives brief explanations that to some extent tie the various strands of work together. Chapters 2–6 provide detailed accounts of proofs of theorems and associated results in optimal recovery theory. Kirov traces the historical development of optimal recovery results for functions in various function classes. Quasi-splines have a structure that is convenient for expressing recovery methods in these classes. The results concentrate on recovery of functions and integrals of functions of one and two variables, and include best methods of recovery in uniform and integral metrics, error bounds for best methods, and comparisons of recovery on different uniform meshes. Tables of optimal quadrature and cubature formulae are included. In the last chapter, the atomar function

$$\lambda(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\cos(tx)\prod_{k=1}^{\infty}\frac{\sin(t/2^k)}{t/2^k}\,dt,$$

which has been applied in solving boundary value problems, is developed as an application of quasi-splines.

The book is written for the approximation theorist, and in particular the specialist in optimal recovery problems. It serves a useful purpose in that much of the material it deals with has not previously appeared in English. Quasi-splines and their basic properties provide an interesting and accessible topic of study for undergraduate level mathematics students. On the other hand, the optimal recovery theorems, which make up a substantial part of the book, are more difficult to study in detail for those without expertise in the field. A number of factors contribute to this difficulty. First, the book is essentially a collection of research papers with limited additional discussion. Whilst Kirov provides an account of the important results pertaining to his particular range of interests as an approximation theorist, he does not attempt to place the work into the broad spectrum of recent developments in optimal recovery methods. Second, access to cited work, almost exclusively in Russian and Bulgarian, appears to be required for some details to be clarified, and third, complicated notation makes reading difficult.

ROBERT CHAMPION

S. THANGAVELU, *Lectures on Hermite and Laguerre Expansions*, Mathematical Notes, Vol. **24**, Princeton University Press, 1993, xv + 195 pp.

The Hermite polynomials H_k and the Laguerre polynomials L_k^α are among the basic special functions of mathematical analysis. When multiplied by their respective weight functions $e^{-x^2/2}$ and $e^{-x/2}x^{\alpha/2}$ and suitably normalized, they become the Hermite functions h_k and Laguerre functions l_k^α, which are orthonormal bases for $L^2(\mathbb{R})$ and $L^2(0,\infty)$, respectively. The Hermite functions have a natural generalization to $\mathbb{R}^n$, namely, $h_\mu(x) = \prod_1^n h_{\mu_j}(x_j)$, which again form an orthonormal basis for $L^2(\mathbb{R}^n)$.

The object of this monograph is a detailed study of the "Fourier analysis" of these bases, encompassing the sorts of problems that have long been studied for Fourier series. For instance, if $f \in L^p$, does the Hermite or Laguerre expansion of f converge to f in the L^p norm? What happens when these expansions are interpreted in terms of summability methods? If T is an operator on L^2 that is diagonal with respect to the Hermite or Laguerre basis, what conditions on the eigenvalues will guarantee that T is well-behaved on other function spaces? Interest in these questions has increased greatly in recent years because of the connection of Hermite and Laguerre functions with the representations of the Heisenberg group, which provides not only motivation for their study but tools for their solution.

To be more specific, the "momentum" operators $Q_j f = i^{-1}\partial f/\partial x_j$ and the "position" operators $Q_j f(x) = x_j f(x)$ on $L^2(\mathbb{R}^n)$ generate a Lie algebra called the Heisenberg algebra. The Heisenberg group $\mathbf{H}_n$ is the corresponding Lie group, and the exponentiated operators $\pi(p,q) = \exp i\Sigma(p_j P_j + q_j Q_j)$ $(p, q \in \mathbb{R}^n)$ determine a unitary representation of $\mathbf{H}_n$. The Gaussian function $e^{-\|x\|^2/2}$ on $\mathbb{R}^n$ plays a special role in this situation for several reasons—for example, it is the ground state of the harmonic oscillator and the extremal function for the uncertainty inequality—and hence the Hermite basis $\{h_\mu\}$ derived from it is of particular interest. Moreover, the matrix elements of π in this basis, $\Phi_{\mu\nu}(p,q) = \langle\pi(p,q)h_\mu, h_\nu\rangle$, turn out to give another orthonormal basis for $L^2(\mathbb{R}^{2n})$. The functions $\Phi_{\mu\nu}$ can be neatly expressed in terms of Laguerre functions, and they are also closely related to Hermite functions on $\mathbb{R}^{2n}$ since they are eigenfunctions for the Hermite operator, so they have been dubbed "special Hermite functions." The interplay among all these things is what gives this subject its distinctive flavor.

Somewhat more than half of the book is devoted to the detailed study of Hermite expansions and special Hermite expansions; the results are then used together with some transplantation theorems to derive information about Laguerre expansions. Much of this is an exposition of the author's own research. It provides an intriguing display of the concepts and techniques of modern Fourier analysis in a setting that has deep classical roots but still deserves further exploration.

GERALD B. FOLLAND

C. DE BOOR, K. HÖLLIG, AND S. D. RIEMENSCHNEIDER, *Box Splines*, Applied Mathematical Sciences, Vol. **98**, Springer-Verlag, 1993, xvii + 200 pp.

Box splines are a natural extension of cardinal splines. A box spline is a compactly supported smooth piecewise polynomial function. Such functions provide an efficient tool for the approximation of curves and surfaces and other smooth functions. Box splines give rise to a beautiful mathematical theory that is much richer than the univariate case. The richness of the box spline theory is due to the complexity of constructing smooth piecewise polynomials on polyhedral cells. On the other hand, it is this richness that allows box splines to have widespread applications.

The book under review, written by three pioneering mathematicians in this area, is the first book to give a complete account of the basic theory of box splines. The authors have not only organized the available material in a cohesive way, but also provide simple and complete proofs in many cases. A large number of illustrations in the book makes its reading enjoyable.

The book begins with a comprehensive description of the box spline and then discusses its various aspects. Given an $s \times n$ real matrix Ξ, the box spline M_Ξ associated with Ξ is defined to be the distribution given by the rule $\phi \mapsto \int_{[0,1)^n}\phi(\Xi t)\,dt$ for $\phi \in C(\mathbb{R}^s)$. In Chapter I various equivalent definitions of the box spline are given. Basic properties of the box spline are derived, and several detailed examples and illustrations are provided to support the mathematical discussion.

The main body of the book falls into three categories: (1) algebraic theory of box splines; (2) approximation and interpolation by box splines, and (3) wavelets and subdivision schemes induced by box splines.

The algebraic theory of box splines is established in Chapter II and further developed in Chapter VI. The box spline M_Ξ is a piecewise polynomial function. These polynomials form a finite dimensional space $D(\Xi)$. The space $D(\Xi)$ can be described as the joint kernel of certain linear partial differential operators with constant coefficients. If each differential operator is replaced by its corresponding difference operator, then the joint kernel $\Delta(\Xi)$ of the resulting difference operators is a linear space of sequences on $\mathbb{Z}^s$. The structure of